

Homework 2

This assignment is due by **11:59pm on Friday 9/13** on gradescope. You should start a new page for each problem (if a problem has multiple parts, the parts can be completed on the same page). After completing the assignment, take (nonblurry) photos/scans of your solutions, save them as a single pdf, and upload your file to gradescope (there are many free apps that allow you to scan and save documents as a pdf; e.g. camscanner). If you have not used gradescope before, see [gradescope.com/help](https://www.gradescope.com/help).

You may collaborate with your peers on homework assignments and to discuss points of confusion with the instructor and/or TAs. However, you must write your own solutions independently and you may not look at or copy any one else's solutions. You must show all of your work to receive full credit.

1. True/False. If a statement is true, prove it. If it is false, provide a counter example.

- (a) If $AB = AC$, then $B = C$.
- (b) If $AB = AC$ and A is invertible, then $B = C$.
- (c) If a matrix has a column of zeroes, then it is not invertible
- (d) Any two elimination matrices (of the same size) commute with each other.

2. Show that the inverse of symmetric matrix is symmetric.

3. For what values of c is $A = \begin{bmatrix} 2 & c & c \\ c & c & c \\ 8 & 7 & c \end{bmatrix}$ invertible?

4. Let $A = \begin{bmatrix} 2 & 1 & 0 \\ 2 & 0 & 2 \\ 6 & 4 & -1 \end{bmatrix}$.

- (a) Use Gauss-Jordan elimination to find the inverse of A .

- (b) Use the inverse matrix to solve the system $A\mathbf{x} = \mathbf{b}$, where $\mathbf{b} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$.

5. Use the LU factorization

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 2 & 0 & 2 \\ 6 & 4 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 3 & -1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 \\ 0 & -1 & 2 \\ 0 & 0 & 1 \end{bmatrix} = LU$$

to solve the system $A\mathbf{x} = \mathbf{b}$, where $\mathbf{b} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$.

6. Let A be a 3×3 matrix and consider the following elimination and permutation matrices.

$$E_{13} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad E_{21} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad P_{13} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

- (a) Describe, in order, the row operations performed on A to obtain the matrix $E_{13}E_{21}P_{13}A$.

- (b) Let's verify your answer to part (a) in an example. Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$. First compute $E_{13}E_{21}P_{13}A$. Next, perform the row operations to A that you found in part (a). Finally, show that both computations yield the same matrix.

7. Let $A = \begin{bmatrix} 2 & 1 & 0 & -1 \\ 2 & 4 & 2 & 1 \\ 6 & 3 & 2 & -2 \\ 2 & 1 & 2 & 0 \end{bmatrix}$

- (a) Use Gaussian elimination to put A in upper triangular form U and write down the corresponding elimination matrices. Express U as a product of A and the elimination matrices.
- (b) Explain why the system $A\mathbf{x} = \mathbf{b}$ can not have a unique solution for all $\mathbf{b} \in \mathbb{R}^4$.
- (c) Use the elimination matrices from part (a) to find an LU factorization of A .

- (d) Let $\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$. Find conditions on the entries of $\mathbf{b}$ so that the system $A\mathbf{x} = \mathbf{b}$ has a solution.

- (e) Find a vector $\mathbf{b} \in \mathbb{R}^4$ such that $A\mathbf{x} = \mathbf{b}$ has no solution.